10

NUMBER AND ALGEBRA

EQUATIONS

Equations are extremely important for solving the same problem many times with different values. In animation and film-making, programmers use complex equations to make action figures move like real people. Most modern appliances are now smart devices with responses that rely on the use of equations.

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Chapter outline

		Working mathematically				
10.01	One-step equations	U	F		R	C
10.02	Two-step equations	U	F		R	C
10.03	Equations with variables on both sides	U	F		R	C
10.04	Equations with brackets	U	F		R	C
10.05	Simple quadratic equations $x^2 = c^{\#}$	U	F		R	C
10.06	Equation problems	U	F	PS	R	C
10.07	Extension: Formulas and equations*	U	F	PS	R	C

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

***Year 9 Extension, Stage 5.2**

Wordbank

backtracking method A method of solving equations by 'undoing', or performing inverse (opposite) operations in reverse order

balancing method A method of solving equations by using the same operations on both sides of the equation

equation A mathematical statement that 2 quantities are equal, involving algebraic expressions and an equals sign (=)

inverse operation An opposite used in solving an equation, for example, the inverse operation of multiplying is adding

solve (an equation) To find the value of an unknown variable in an equation

solution The answer to an equation or problem, the correct value(s) of a variable that makes an equation true

substitute To replace a variable with a number

variable A quantity that can take on different values, represented by a symbol such as a letter of the alphabet

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:


Solving equations

- solve a variety of linear equations, including those with variables on both sides, and brackets
- solve problems by using equations
- solve simple quadratic equations of the form $x^2 = c$
- (EXTENSION, STAGE 5.2) solve equations involving formulas to solve problems

SkillCheck ANSWERS ON P. 576

1 Find the number represented by $\square$ to make each equation true.

a $\square + 3 = 10$ **b** $4 \times \square = 28$ **c** $12 - \square = 4$

d $\square \div 3 = 9$ **e** $9 + \square = 15$ **f** $\square - 7 = 16$

g $\square \times 6 = 30$ **h** $24 \div \square = 8$ **i** $10 + \square = 20$

j $\square - 5 = 0$ **k** $\frac{1}{2} \times \square = 4$ **l** $\square \div 5 = 4$

2 Evaluate each expression.

a $5 - 8$ **b** $-8 + 3$ **c** $3 \times (-4)$

d $-6 \div 3$ **e** $-2 + 5$ **f** $-2 - 9$

g $-4 \times (-9)$ **h** $-12 \div (-4)$ **i** $-3 - 7$

j $80 \div (-10)$ **k** $6 - 15$ **l** -8×9

3 If $d = 7$, evaluate:

a $2d + 3$ **b** $3d - 4$ **c** $\frac{d+1}{2}$ **d** $18 - 2d$

4 If $x = -3$, evaluate:

a $5x + 9$ **b** $4 - x$ **c** $\frac{x-3}{-1}$ **d** $2x - 8$

5 Simplify each expression.

a $3n - 2n$ **b** $8x + 3x$ **c** $5r - r$ **d** $6t + 2t$

6 Expand each expression.

a $2(m + 3)$ **b** $3(x - 2)$ **c** $4(k + 7)$

d $5(d - 1)$ **e** $6(a - 3)$ **f** $4(b + 8)$

g $5(2q - 6)$ **h** $9(5j + 1)$

7 What is the opposite operation to:

a adding? **b** dividing?

c multiplying? **d** subtracting?

One-step equations

10.01

An **equation** is a statement involving a variable (such as x), numbers and an equals (=) sign, for example, $\mathbf{2x - 5 = 11}$. In Year 7 we learnt how to **solve** equations, that is, to find the value of the variable that makes the equation true. This value is called the **solution** to the equation.

There are 2 algebraic methods for solving equations.

The **balancing method** involves representing both sides of an equation as balance scales, and solving the equation by performing the same operation on both sides to keep it 'balanced'.

The **backtracking method** involves using a flow chart to show what operations are performed on the variable (say, x) to create the equation, then using a reverse flow chart to undo each operation by performing the inverse (opposite) operation in reverse order.

WS Equations 1
WS One-step equations
Equations dominoes
One-step equations
WS Guess-and-check

Solving equations

To solve an equation, aim to have the variable (such as x) on one side of the equation and a number on the other side, in the form:

$$x = \_\_\_\_$$

Check your solution by substituting it back into the equation.

Example 1

Solve each equation.

a $m + 4 = 12$ **b** $3y = 18$ **c** $k - 3 = 5$ **d** $\frac{t}{8} = 6$

Solution

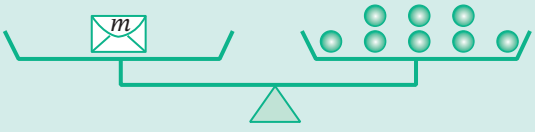
a **Method 1: The balancing method**

$m + 4 = 12$

This equation can be represented on a balance scale:

$m + 4 - 4 = 12 - 4$

Subtracting 4 from both sides.

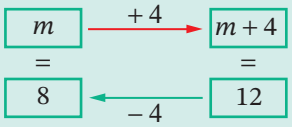


$m = 8$

Check: $8 + 4 = 12$.

Method 2: The backtracking method

$m + 4 = 12$ — This equation can be represented on a flowchart, starting at m and ending at 12:



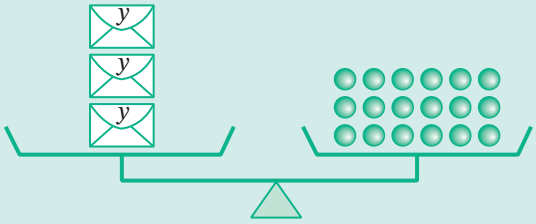
$m = 12 - 4$ — Undo '+ 4' by subtracting 4.

Starting at 12 on the flowchart and ending at 8.

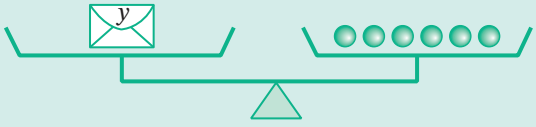
$m = 8$ — Check: $8 + 4 = 12$.

b **Method 1: The balancing method**

$3y = 18$



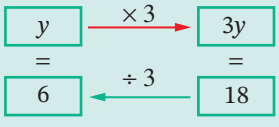
$\frac{3y}{3} = \frac{18}{3}$ — Dividing both sides by 3.



$y = 6$ — Check: $3 \times 6 = 18$

Method 2: The backtracking method

$3y = 18$



$y = \frac{18}{3}$ — Undo '× 3' in $3y$ by dividing by 3.

$y = 6$ — Check: $3 \times 6 = 18$

c **Method 1: The balancing method**

$k - 3 = 5$

$k - 3 + 3 = 5 + 3$ — Adding 3 to both sides.

$k = 8$ — Check: $8 - 3 = 5$

Method 2: The backtracking method

$k - 3 = 5$

$k = 5 + 3$ — Undo '− 3' by adding 3.

$k = 8$ — Check: $8 - 3 = 5$

d **Method 1: The balancing method**

$\frac{t}{8} = 6$

$\frac{t}{8} \times 8 = 6 \times 8$ Multiplying both sides by 8.

$t = 48$ Check: $\frac{48}{8} = 6$

Method 2: The backtracking method

$\frac{t}{8} = 6$

$t = 6 \times 8$ Undo '÷ 8' by multiplying by 8.

$t = 48$ Check: $\frac{48}{8} = 6$

These are all 'one-step' equations because they require only one step or one inverse operation to solve.

Inverse operations

Operation	Inverse operation
+	−
−	+
×	÷
÷	×

EXERCISE 10.01 ANSWERS ON P. 576

One-step equations UFRC

EXAMPLE 1

1 Solve each equation, showing the working.

a $x + 5 = 12$ **b** $y + 13 = 36$ **c** $m + 9 = 21$ **d** $k + 27 = 54$

e $k + 6 = 2$ **f** $s + 3 = 1$ **g** $n + 9 = -4$ **h** $p + 17 = -3$

i $m + 8 = 0$ **j** $g + 4.5 = 20$ **k** $a + 1\frac{3}{4} = 12$ **l** $k + 2.7 = -5$

2 Solve each equation, showing the working.

a $t - 11 = 40$ **b** $w - 7 = 4$ **c** $a - 5 = 2$ **d** $j - 23 = 51$

e $q - 12 = 17$ **f** $f - 42 = 68$ **g** $x - 9 = 24$ **h** $g - 10 = 1$

i $j - 3 = -5$ **j** $w - 9 = -2$ **k** $m - 12 = -27$ **l** $d - 1 = -1$

m $y - 17 = 3.9$ **n** $n - 2.1 = 5.9$ **o** $b - \frac{4}{5} = \frac{1}{4}$ **p** $s - 2\frac{1}{3} = 4\frac{1}{2}$

3 Solve each equation, showing the working.

a $6x = 24$ **b** $4g = 16$ **c** $10y = 90$ **d** $5b = 45$

e $12d = 108$ **f** $9c = 81$ **g** $23p = 115$ **h** $2m = 26$

i $5x = -25$ **j** $7q = -42$ **k** $-3c = 54$ **l** $-15x = -120$

m $4p = 37$ **n** $8r = 18$ **o** $3q = -10$ **p** $9w = -12$

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4 Solve each equation, showing the working.

a $\frac{s}{3} = 5$ **b** $\frac{m}{7} = 6$ **c** $\frac{d}{10} = 12$ **d** $\frac{w}{2} = 29$

e $\frac{a}{3} = -4$ **f** $\frac{f}{-2} = 9$ **g** $\frac{r}{5} = -3$ **h** $\frac{m}{-7} = 1$

i $\frac{x}{-8} = -6$ **j** $\frac{t}{2} = \frac{3}{4}$ **k** $\frac{n}{-4} = -4$ **l** $\frac{x}{3} = -\frac{1}{6}$

5 Solve $3x = -12$. Select the correct answer **A**, **B**, **C** or **D**.

A $x = -9$ **B** $x = -36$ **C** $x = -4$ **D** $x = -15$

6 Write a one-step equation whose solution is: R C

a $m = 4$ **b** $x = 0$ **c** $r = 7$ **d** $d = -5$

10.02 Two-step equations

Two-step equations

Solving equations

Solving equations by balancing

Solving equations by backtracking

Solving equations using diagrams

Backtracking

Solving linear equations

Linear equation solver

Equations writing activity

Two-step equations require 2 steps or 2 inverse operations to solve.

Example 2

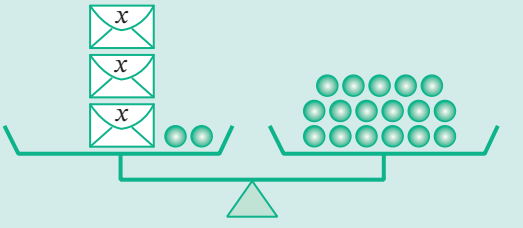
Solve each equation.

a $3x + 2 = 17$ **b** $\frac{k}{5} + 4 = 7$

Solution

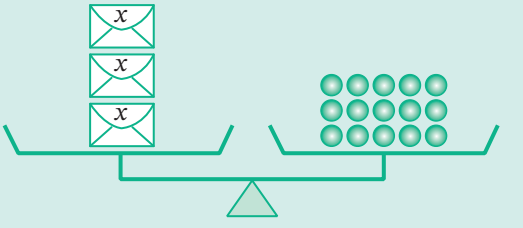
a **Method 1: The balancing method**

$3x + 2 = 17$



$3x + 2 - 2 = 17 - 2$

Step 1: Subtracting 2 from both sides.



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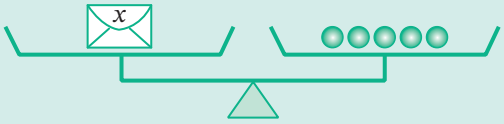
$3x = 15$

$\frac{3x}{3} = \frac{15}{3}$

Step 2: Dividing both sides by 3.

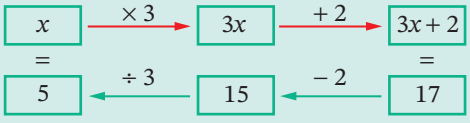


$x = 5$

Check: $3 \times 5 + 2 = 17$

Method 2: The backtracking method

$3x + 2 = 17$



$3x = 17 - 2$

Step 1: Undo '+ 2' by subtracting 2.

$x = \frac{15}{3}$

Step 2: Undo '× 3' in 3x by dividing by 3.

$x = 5$

Check: $3 \times 5 + 2 = 17$

b **Method 1: The balancing method**

$\frac{k}{5} + 4 = 7$

$\frac{k}{5} + 4 - 4 = 7 - 4$

Step 1: Subtracting 4 from both sides.

$\frac{k}{5} = 3$

$\frac{k}{5} \times 5 = 3 \times 5$

Step 2: Multiplying both sides by 5.

$k = 15$

Check: $\frac{15}{5} + 4 = 3 + 4 = 7$

Method 2: The backtracking method

$\frac{k}{5} + 4 = 7$

$\frac{k}{5} = 7 - 4$

Step 1: Undo '+ 4' by subtracting 4.

$\frac{k}{5} = 3$

$k = 3 \times 5$

Step 2: Undo '÷ 5' by multiplying by 5.

$k = 15$

Check: $\frac{15}{5} + 4 = 3 + 4 = 7$

Two-step equations

Example 3

Solve each equation.

a $\frac{2h}{3} = 4$ **b** $\frac{x+1}{2} = 7$

Solution

a **Method 1: The balancing method**

$\frac{2h}{3} = 4$

$\frac{2h}{3} \times 3 = 4 \times 3$ *Step 1*: Multiplying both sides by 3.

$2h = 12$

$\frac{2h}{2} = \frac{12}{2}$ *Step 2*: Dividing both sides by 2.

$h = 6$ Check: $\frac{2 \times 6}{3} = 4$

Method 2: The backtracking method

$\frac{2h}{3} = 4$

$2h = 4 \times 3$ *Step 1*: Undo '÷ 3' by multiplying by 3.

$2h = 12$

$h = \frac{12}{2}$ *Step 2*: Undo '× 2' in 2h by dividing by 2.

$h = 6$ Check: $\frac{2 \times 6}{3} = 4$

b **Method 1: The balancing method**

$\frac{x+1}{2} = 7$

$\frac{x+1}{2} \times 2 = 7 \times 2$ *Step 1*: Multiplying both sides by 2.

$x + 1 = 14$

$x + 1 - 1 = 14 - 1$ *Step 2*: Subtracting 1 from both sides.

$x = 13$ Check: $\frac{13+1}{2} = 7$

Method 2: The backtracking method

$\frac{x+1}{2} = 7$

$x + 1 = 7 \times 2$ — *Step 1*: Undo '÷ 2' by multiplying by 2.

$x + 1 = 14$

$x = 14 - 1$ — *Step 2*: Undo '+ 1' by subtracting 1.

$x = 13$ — Check: $\frac{13+1}{2} = 7$

EXERCISE 10.02 ANSWERS ON P. 576

Two-step equations U F R C

1 Check which equation has $p = -2$ as its solution. Select the correct answer **A**, **B**, **C** or **D**.

A $2 + 3p = 8$ **B** $5p - 2 = 8$ **C** $-4p + 16 = 8$ **D** $-3p + 2 = 8$

EXAMPLE 2

2 Solve each equation, showing all steps. Remember to check your answers.

a $2m + 7 = 19$ **b** $3x - 5 = 13$ **c** $5k + 12 = 52$

d $6w - 17 = 19$ **e** $4h + 21 = 9$ **f** $8d - 5 = -27$

g $-2x + 18 = 4$ **h** $-3a - 9 = -6$ **i** $14c + 2 = 37$

j $12p - 10 = 86$ **k** $9y + 11 = 101$ **l** $2r - 15 = 16$

3 Solve each equation.

a $\frac{m}{2} + 4 = 13$ **b** $\frac{c}{5} + 9 = 12$ **c** $\frac{d}{7} + 2 = 8$

d $\frac{k}{3} + 15 = 6$ **e** $\frac{h}{2} + 7 = 17$ **f** $\frac{x}{4} + 2 = -4$

g $\frac{n}{5} - 9 = 4$ **h** $\frac{t}{3} - 6 = -2$ **i** $\frac{a}{8} - 2 = -6$

j $\frac{x}{6} - 16 = -13$ **k** $\frac{v}{15} - 2 = 4$ **l** $\frac{b}{3} - 4 = 3$

4 This is Liam's incorrect solution for $7x + 5 = 13$.

$7x + 5 = 13$

$7x = 13 - 5$ **A**

$7x = 8$ **B**

$x = \frac{8}{7}$ **C**

$x = 1\frac{1}{8}$ **D**

In which line was the error made? Select **A**, **B**, **C** or **D**. R

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EXAMPLE 3

5 Solve each equation.

a $\frac{3x}{2} = 9$ **b** $\frac{15x}{9} = 15$ **c** $\frac{2x}{3} = 8$ **d** $\frac{3N}{5} = 3$

e $\frac{6N}{7} = 18$ **f** $\frac{5B}{2} = -10$ **g** $\frac{3x}{4} = -3$ **h** $\frac{2x}{5} = -4$

i $\frac{5m}{2} = 11$ **j** $\frac{-x}{3} = 8$ **k** $\frac{-4x}{5} = 1$ **l** $\frac{-2x}{3} = -10$

6 Check which value of k is the solution to $\frac{k-12}{9} = 6$. Select **A**, **B**, **C** or **D**.

A $k = 27$ **B** $k = 42$ **C** $k = 66$ **D** $k = 162$

7 Solve each equation.

a $\frac{x+1}{3} = 2$ **b** $\frac{x-3}{2} = 3$ **c** $\frac{N+2}{5} = 1$ **d** $\frac{N-3}{4} = 5$

e $\frac{N+8}{5} = 6$ **f** $\frac{x+1}{2} = -2$ **g** $\frac{k-5}{2} = -5$ **h** $\frac{m+2}{3} = 11$

8 Write 4 equations, one of each type shown in questions **2**, **3**, **5** and **7**, that have the solution $p = 2$. R C

10.03 Equations with variables on both sides

WS Equations with unknowns on both sides

Equations with unknowns on both sides

Solving equations

Formally solving equations

For equations with variables on both sides, such as $3x + 4 = 2x + 7$, we can only use the **balancing method**, not the backtracking method.

Equations with variables on both sides

Perform operations on both sides to move:

- all the variables onto one side of the equation
- all the numbers onto the other side of the equation.

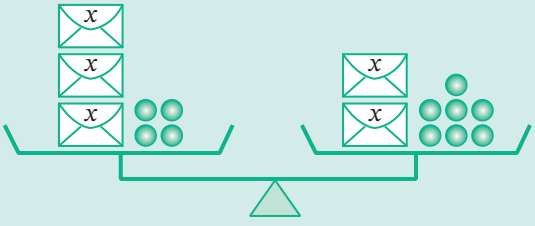
Example 4

Solve each equation.

a $3x + 4 = 2x + 7$ **b** $5n - 3 = 2n - 15$ **c** $2d - 8 = -5d - 71$

Solution

a $3x + 4 = 2x + 7$



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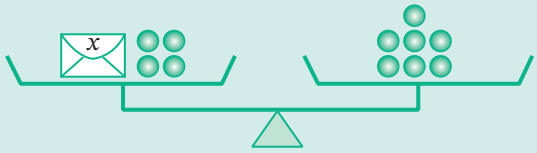
$3x + 4 - 2x = 2x + 7 - 2x$ — Subtracting $2x$ from both sides to remove it from the RHS (right-hand side).

$x + 4 = 7$



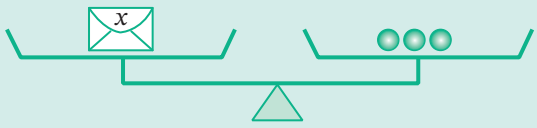
$x + 4 - 4 = 7 - 4$ — Subtracting 4 from both sides to remove it from the LHS (left-hand side).

$x = 3$

Check: LHS $= 3 \times 3 + 4 = 13$

RHS $= 2 \times 3 + 7 = 13$

LHS = RHS.



b $5n - 3 = 2n - 15$

$5n - 3 - 2n = 2n - 15 - 2n$ — Subtracting $2n$ from both sides to remove it from the RHS.

$3n - 3 = -15$ — Now this is a two-step equation.

$3n - 3 + 3 = -15 + 3$ — Adding 3 to both sides to remove it from the LHS.

$3n = -12$ — Now this is a one-step equation.

$\frac{3n}{3} = \frac{-12}{3}$ — Dividing both sides by 3.

$n = -4$

Check:

LHS $= 5 \times (-4) - 3 = -23$

RHS $= 2 \times (-4) - 15 = -23$

LHS = RHS

c $2d - 8 = -5d - 71$

$2d - 8 + 5d = -5d - 71 + 5d$ — Adding $5d$ to both sides to remove it from the RHS.

$7d - 8 = -71$ — Now this is a two-step equation.

$7d - 8 + 8 = -71 + 8$ — Add 8 to both sides to remove it from the LHS.

$7d = -63$ — Now this is a one-step equation.

$\frac{7d}{7} = \frac{-63}{7}$ — Dividing both sides by 7.

$d = -9$

Check:

LHS $= 2 \times (-9) - 8 = -26$

RHS $= -5 \times (-9) - 71 = -26$

LHS = RHS.

EXERCISE 10.03 ANSWERS ON P. 576

Equations with variables on both sides U F R C

EXAMPLE 4

1 What is the first step in solving $3x - 4 = x + 5$? Select the correct answer **A**, **B**, **C** or **D**. R C

A subtract x from both sides **B** add x to both sides

C add $3x$ to both sides **D** subtract 4 from both sides

2 Solve $10m - 2 = 6m$. Select **A**, **B**, **C** or **D**.

A $-\frac{1}{2}$ **B** $\frac{1}{8}$ **C** $\frac{1}{2}$ **D** 2

3 Solve each equation, showing all steps. Remember to check your answers.

a $3a + 6 = a + 18$ **b** $2k + 4 = k + 8$ **c** $3x + 6 = 2x + 9$

d $6p + 4 = p + 19$ **e** $6n + 5 = 2n + 17$ **f** $5q + 6 = 4q + 12$

g $5y + 14 = 3y + 14$ **h** $4a + 7 = 2a + 21$ **i** $2r + 9 = r + 15$

4 Solve $4x - 5 = 2x + 7$. Select **A**, **B**, **C** or **D**.

A $x = 1$ **B** $x = 2$ **C** $x = 4$ **D** $x = 6$

5 Solve each equation.

a $5a - 3 = 2a + 6$ **b** $6x - 2 = 3x + 6$

c $3d - 6 = d - 4$ **d** $8p - 15 = 3p - 10$

e $3m + 5 = -m + 26$ **f** $4x + 6 = -6x + 56$

g $4x + 3 = -2x + 7$ **h** $2r + 18 = -5r + 11$

i $3y - 12 = -y + 6$

6 Here is Bree's solution for $3x - 4 = -2x + 8$.

$3x - 4 = -2x + 8$

$3x + 2x - 4 = 8$ Line 1

$5x - 4 = 8$ Line 2

$5x = 8 - 4$ Line 3

$5x = 4$ Line 4

$x = \frac{5}{4}$ Line 5

In which lines did Bree make mistakes? Select **A**, **B**, **C** or **D**. R

A Lines 1 and 3 **B** Lines 2 and 5 **C** Lines 1 and 4 **D** Lines 3 and 5

7 Solve each equation.

a $3d + 4 = d$ **b** $10k = 12 - 8k$ **c** $5 - p = p + 9$

d $7 + x = 6x + 22$ **e** $6k - 11 = k$ **f** $3m + 20 = 7m - 2$

g $4t = 12 - 4t$ **h** $8j - 17 = 10j$ **i** $6 - 3q = 8 - q$

8 Write an equation with x on both sides that has the solution $x = 7$. R C

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Equations with brackets 10.04

Equations with brackets (grouping symbols)

Expand the expressions and then solve as usual.

Example 5

Solve each equation.

a $3(x + 5) = 9$ **b** $2(3 - y) = 4y + 4$ **c** $5(r - 3) = 2(r + 6)$

Solution

a $3(x + 5) = 9$

$3x + 15 = 9$ Expanding the expression to make it a two-step equation.

Can you think of another way to solve this equation without expanding?

$3x + 15 - 15 = 9 - 15$ Subtracting 15 from both sides.

$3x = -6$

$\frac{3x}{3} = \frac{-6}{3}$ Dividing both sides by 3.

$x = -2$ Check: $3(-2 + 5) = 3 \times 3 = 9$

b $2(3 - y) = 4y + 4$

$6 - 2y = 4y + 4$ Expanding the brackets.

$6 - 2y - 4y = 4y + 4 - 4y$ Subtracting $4y$ from both sides to remove it from the RHS.

$6 - 6y = 4$ Now this is a two-step equation.

$6 - 6y - 6 = 4 - 6$ Subtracting 6 from both sides to remove it from the LHS.

$-6y = -2$

$\frac{-6y}{-6} = \frac{-2}{-6}$ Dividing both sides by (-6).

$y = \frac{1}{3}$ Check:

$\text{LHS} = 2 \times \left(3 - \frac{1}{3}\right) = 5\frac{1}{3}$

$\text{RHS} = 4 \times \frac{1}{3} + 4 = 5\frac{1}{3}$

LHS = RHS

c $5(r-3)=2(r+6)$

$5r-15=2r+12$	Expanding both sides.
$5r-15-2r=2r+12-2r$	Subtract $2r$ from both sides to remove it from the RHS.
$3r-15=12$	Now this is a two-step equation.
$3r-15+15=12+15$	Add 15 to both sides to remove it from the LHS.
$3r=27$	
$\frac{3r}{3}=\frac{27}{3}$	Dividing both sides by 3.
$r=9$	Check:
	LHS $=5\times(9-3)=5\times6=30$
	RHS $=2\times(9+6)=2\times15=30$
	LHS = RHS

EXERCISE 10.04 ANSWERS ON P. 576

Equations with brackets UFRC

EXAMPLE 5

1 Solve each equation, showing all steps. Remember to check your answers.

a	$2(x+3)=8$	**b**	$3(m+2)=18$	**c**	$5(k+1)=35$
d	$4(p+4)=32$	**e**	$10(j+2)=50$	**f**	$7(d+4)=63$
g	$3(x-4)=15$	**h**	$3(x-4)=12$	**i**	$5(x-1)=10$
j	$8(x-5)=24$	**k**	$4(k-5)=28$	**l**	$2(q-7)=26$

2 Solve each equation.

a	$2(6-x)=6$	**b**	$3(4-p)=24$	**c**	$2(1-q)=8$
d	$3(16-h)=15$	**e**	$2(4+r)=-8$	**f**	$3(e-16)=-15$
g	$5(z-3)=-10$	**h**	$-7(y+2)=35$	**i**	$9(6-a)=0$
j	$5(d+3)=-20$	**k**	$-2(3-k)=12$	**l**	$4(6-c)=-16$

3 Solve $2(12-3x)=30$. Select the correct answer **A**, **B**, **C** or **D**.

A $x=-2$ **B** $x=-1$ **C** $x=1$ **D** $x=2$

4 Solve each equation.

a	$6(x+1)=3x+3$	**b**	$2(r+2)=r+5$
c	$3(p+3)=2p+2$	**d**	$2(x+1)=x$
e	$5(p-2)=3p$	**f**	$4(e-2)=2e+4$
g	$5(y+2)=3y+12$	**h**	$3(a-5)=5-2a$
i	$3(2w+1)=3w-15$		

5 Here is Ryan's solution for $7(h-4)=3h+20$.

$$7(h-4)=3h+20$$
$$7h-4=3h+20 \quad \text{Line A}$$
$$7h-4-3h=3h+20-3h$$
$$4h-4=20 \quad \text{Line B}$$
$$4h-4+4=20+4$$
$$4h=24 \quad \text{Line C}$$
$$\frac{4h}{4}=\frac{24}{4}$$
$$h=6 \quad \text{Line D}$$

In which line did Ryan make a mistake? Select **A**, **B**, **C** or **D**. R C

6 Solve each equation.

a $5(x-1)=4(x+2)$ **b** $6(x+2)=4(x+6)$ **c** $4(k+3)=3(k-2)$

d $7(y+2)=4(y+5)$ **e** $3(v+2)=2(v+5)$ **f** $4(x-2)=2(x+7)$

g $3(p+1)=5(p-1)$ **h** $3(2s+1)=5(s+2)$ **i** $5(d+4)=2(2d-1)$

7 Write an equation involving brackets whose solution is: R C

a $c=5$ **b** $k=-1$ **c** $n=2$ **d** $q=-4$

10.04

Mental skills 10: Maths without calculators ANSWERS ON P. 577

Multiplying and dividing by 5, 15, 25 and 50

It is easier to multiply or divide a number by 10 than by 5. So whenever we multiply or divide a number by 5, we can double the 5 (to make 10) and then adjust the first number.

1 Study each example.

a To multiply by 5, halve the number, then multiply by 10.

$$18\times5=18\times\frac{1}{2}\times10 \quad (\text{or } 9\times2\times10)$$
$$=9\times10$$
$$=90$$

b To multiply by 50, halve the number, then multiply by 100.

$$26\times50=26\times\frac{1}{2}\times100 \quad (\text{or } 13\times2\times100)$$
$$=13\times100$$
$$=1300$$

c To multiply by 25, quarter the number, then multiply by 100.

$$44\times25=44\times\frac{1}{4}\times100 \quad (\text{or } 11\times4\times25)$$
$$=11\times100$$
$$=1100$$

d To multiply by 15, halve the number, then multiply by 30.

$$8\times15=8\times\frac{1}{2}\times30 \quad (\text{or } 4\times2\times15)$$
$$=4\times30$$
$$=120$$

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e To divide by 5, divide by 10 and double the answer. We do this because there are two 5s in every 10.

$140 \div 5 = 140 \div 10 \times 2$
$= 14 \times 2$
$= 28$

f To divide by 50, divide by 100 and double the answer. This is because there are two 50s in every 100.

$400 \div 50 = 400 \div 100 \times 2$
$= 4 \times 2$
$= 8$

g To divide by 25, divide by 100 and multiply the answer by 4. This is because there are four 25s in every 100.

$600 \div 25 = 600 \div 100 \times 4$
$= 6 \times 4$
$= 24$

h To divide by 15, divide by 30 and double the answer. This is because there are two 15s in every 30.

$240 \div 15 = 240 \div 30 \times 2$
$= 8 \times 2$
$= 16$

2 Now evaluate each expression.

a	32×5	**b**	14×5	**c**	48×5	**d**	18×50
e	52×50	**f**	36×25	**g**	28×5	**h**	12×25
i	12×15	**j**	22×35	**k**	$90 \div 5$	**l**	$170 \div 5$
m	$230 \div 5$	**n**	$1300 \div 50$	**o**	$900 \div 50$	**p**	$300 \div 25$
q	$1000 \div 25$	**r**	$360 \div 45$	**s**	$210 \div 15$	**t**	$360 \div 15$

Investigation

Solving $x^2 = c$

1 What is the inverse operation of 'squaring'?

2 The equation $x^2 = 9$ has 2 solutions. What are the 2 numbers, x, which when squared give 9?

3 What are the solutions for each equation? What is the pattern found in the answers?

a $x^2 = 25$ **b** $x^2 = 100$ **c** $x^2 = 1$

4 Study this example:

$x^2 = 49$
$x = \pm\sqrt{49}$ which means $x = \sqrt{49}$ or $-\sqrt{49}$
$= \pm 7$ which means $x = 7$ or $x = -7$.

Check: When $x = 7$, $x^2 = 7^2 = 49$

When $x = -7$, $x^2 = (-7)^2 = 49$

Use the same method to solve each equation and check your answers:

a $x^2 = 81$ **b** $x^2 = 64$

Simple quadratic equations $x^2 = c$#

10.05

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

An equation involving a variable squared, such as $x^2 = 25$ or $3x^2 - 4 = 7$, is called a **quadratic equation**. In this section, we will solve simple quadratic equations of the type $x^2 = c$, where c is a number.

Example 6

Solve each quadratic equation.

a $x^2 = 36$

b $x^2 = 121$

c $x^2 = 40$, writing the solution correct to one decimal place

d $x^2 = 83$, writing the solution as a surd

Solution

a
$x^2 = 36$ — Finding the square root of both sides.
$x = \pm\sqrt{36}$ — The equation has 2 solutions, $x = 6$ or $x = -6$.
$= \pm 6$

b
$x^2 = 121$ — Finding the square root of both sides.
$x = \pm\sqrt{121}$ — The equation has solutions $x = 11$ or $x = -11$.
$= \pm 11$

c
$x^2 = 40$ — The equation has solutions $x \approx 6.3$ or $x \approx -6.3$.
$x = \pm\sqrt{40}$
$= \pm 6.3345\ldots$
$\approx \pm 6.3$

d
$x^2 = 83$ — Leaving the answer as a surd.
$x = \pm\sqrt{83}$ — $x = \sqrt{83}$ or $-\sqrt{83}$

Simple quadratic equations $x^2 = c$

The simple quadratic equation $x^2 = c$ (where c is a positive number) has 2 solutions,

$$x = \pm\sqrt{c}$$

(which means $x = \sqrt{c}$ or $x = -\sqrt{c}$).

EXERCISE 10.05 ANSWERS ON P. 577

Simple quadratic equations $x^2 = c$ U F R C

EXAMPLE 6

1 Solve each quadratic equation.

a $x^2 = 81$ **b** $x^2 = 144$ **c** $x^2 = 1$

d $x^2 = 169$ **e** $m^2 = 5041$ **f** $u^2 = 1849$

2 Solve each quadratic equation, writing the solution correct to 2 decimal places.

a $x^2 = 13$ **b** $x^2 = 54$ **c** $x^2 = 88$

d $t^2 = 129$ **e** $h^2 = 946$ **f** $z^2 = 527$

3 Solve each quadratic equation, writing the solution as a surd.

a $x^2 = 41$ **b** $x^2 = 30$ **c** $x^2 = 48$

d $a^2 = 126$ **e** $p^2 = 75$ **f** $b^2 = 509$

4 Many simple quadratic equations have 2 solutions (one positive, one negative), but there is one simple quadratic equation that has only **one** solution. What is this equation and what is its solution? R

5 The equation $x^2 = -100$ has **no solutions**. Explain why. R C

6 Write 2 more simple quadratic equations that have no solutions. R C

7 Copy and complete: R C

a $x^2 = c$ has 2 solutions if c is _____.

b $x^2 = c$ has no solutions if c is _____.

c $x^2 = c$ has one solution if c is _____.

8 Solve each quadratic equation. R

a $3x^2 = 27$ **b** $2x^2 = 32$ **c** $3x^2 = 75$ **d** $5x^2 - 1 = 19$

e $5x^2 + 1 = 21$ **f** $7 + 3x^2 = 34$ **g** $6x^2 - 8 = 142$ **h** $7x^2 + 9 = 261$

Did you know?

Applications of quadratic equations

Quadratic equations can be more complex, for example, $3x^2 + 4x - 15 = 0$, with applications in many areas:

- the shape of satellite dishes and reflecting telescopes
- the path of a thrown object, such as a ball, shotput or javelin
- the orbits of planets as they move around the Sun.

Research the formula that solves quadratic equations.

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Equation problems

10.06

Solving word problems requiring an equation

- Choose your variable
- Translate the words into an equation
- Solve the equation
- Write a sentence that answers the problem

Example 7

a When a number is doubled and 11 is subtracted, the result is 23. Find the number.

b 5 times a number is the same as 6 more than 3 times the number. What is the number?

Solution

a Let the number be x.

$x \times 2 - 11 = 23$ Translating the words into an equation

$2x - 11 = 23$

Solving the equation:

$2x - 11 + 11 = 23 + 11$ Adding 11 to both sides

$2x = 34$

$\frac{2x}{2} = \frac{34}{2}$ Dividing both sides by 2

$x = 17$ Check: $2 \times 17 - 11 = 23$

The number is 17.

b Let n represent the number.

$5n = 3n + 6$

$5n - 3n = 3n + 6 - 3n$ Subtracting $3n$ from both sides

$2n = 6$

$\frac{2n}{2} = \frac{6}{2}$ Dividing both sides by 2

$n = 3$

The number is 3. Check:

LHS $= 5 \times 3 = 15$

RHS $= 3 \times 3 + 6 = 15$

LHS = RHS

Example 8

Cooper charges for fixing washing machines using the formula: $C = 45h + 70$, where C is the charge in dollars and h is the number of hours the job takes.

Find:

a the charge for a job that takes 3 hours

b the number of hours Cooper worked if the charge is $430.

Solution

a Substitute $h = 3$ into the formula.

$$\begin{aligned} C &= 45h + 70 \\ &= 45 \times 3 + 70 \\ &= 205 \end{aligned}$$

The charge is $205.

b Substitute $C = 430$ into the formula and solve the equation.

$$430 = 45h + 70$$

$$45h + 70 = 430$$

$$45h + 70 - 70 = 430 - 70$$

$$45h = 360$$

$$\frac{45h}{45} = \frac{360}{45}$$

$$h = 8$$

The number of hours Cooper worked is 8.

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EXERCISE 10.06 ANSWERS ON P. 577

Equation problems U F PS R C

EXAMPLE 7

10.06

1 Solve each problem by writing an equation and then solving it. You may use diagrams to help you think about the information. R C

a 4 family tickets for a theme park cost $352. How much does each ticket cost? (Let t represent the price of a family ticket.)

b 20 cans of drink cost $36. How much does each can cost? (Let c represent the cost of one can.)

c A number is tripled and the result is 99. What is the number? (Let x represent the unknown number.)

d A number has 7 added to it and the result is 23. Find the number. (Let n represent the unknown number.)

2 Select the correct equation **A**, **B**, **C** or **D** for each problem. Then use the equation to solve the problem. R C

a If 23 is subtracted from a number, the answer is 79. What is the number?

A $79 + N = 23$ **B** $23N = 79$ **C** $23 - N = 79$ **D** $N - 23 = 79$

b The sum of a number and 18 is 67. What is the number?

A $N + 67 = 18$ **B** $N + 18 = 67$ **C** $18N = 67$ **D** $N - 18 = 67$

c When a number is multiplied by 23, the answer is 1288. What is the number?

A $1288N = 23$ **B** $N + 23 = 1288$ **C** $1288 - N = 23$ **D** $23N = 1288$

d When a number is divided by 14, the answer is 42. What is the number?

A $N - 14 = 42$ **B** $\frac{N}{14} = 42$ **C** $14N = 42$ **D** $14 + N = 42$

e Marika buys a car that costs $13 499. She pays a deposit of $4525. How much does she owe?

A $4525N = 13\,499$ **B** $N - 4525 = 13\,499$

C $N + 4525 = 13\,499$ **D** $N = 13\,499 + 4525$

3 For each problem, write an equation and find the unknown number. R C

a When an unknown number is divided by 5, the result is 15.

b When 12 is subtracted from a number, the answer is 37.

c When a number is subtracted from 52, the answer is 25.

d An unknown number is multiplied by 3, then 8 is subtracted. The result is 40.

e An unknown number is halved, then 7 is added. The result is 13.

f The sum of a number and 4 is doubled. The result is 44.

g The sum of a number and 6 is divided by 3 and the result is 7.

h The product of a number and 5 is decreased by 2. The result is 28.

4 Translate each problem into an equation, then solve the equation to solve the problem. PS R C

a The student council is holding a disco to raise money. Each ticket bought by students raises $20, but the costs of running the disco total $1000. How many tickets must be sold to make a profit of $2000? (Let n stand for the number of tickets sold.)

b In 8 years Kelly's age will be twice what it is now. How old is Kelly now? (Let n stand for Kelly's age now.)

c 8 sheep have the same mass as 3 sheep and one cow. If the cow's mass is 500 kg, what is the mass of one sheep? (Let s stand for the mass of one sheep.)

d If you multiply Liam's favourite number by 3 and add 1, you get the same answer as if you multiplied the number by 5 and took away 11. What is the number? (Let x represent the number.)

e Mr Yen says, 'If you add 15 to my age and multiply by 7, the answer is 483.' How old is Mr Yen? (Let a stand for his age.)

f The area of a rhombus is calculated by multiplying its diagonals and dividing by 2. If a rhombus has an area of 88 cm^2 and one diagonal is 11 cm, what is the length of the other diagonal? (Let d represent the length of the diagonal.)

EXAMPLE 8

5 A temperature in degrees Celsius (°C) can be converted to degrees Fahrenheit (°F) using the formula $F = \frac{9C}{5} + 32$. PS R

a Convert 25°C to °F.

b Convert 108°F to °C.

6 The charge, $\$C$, for hiring a hall for an event is $C = 150 + 2N$, where N stands for the number of people at the event. Find: PS R

a the charge when 225 people are at the event

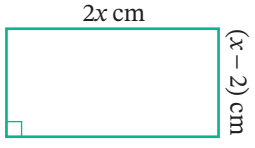
b the number of people at the event when the charge is $394.

7 The cost, $\$y$, of an online ad on a local news website is $y = 0.8w + 3.5$, where w is the number of words in the ad. Find: PS R

a the cost of a 13-word ad

b the number of words in an ad costing $25.90

8 The perimeter of this rectangle is 47 cm. Find: PS R



a the value of x

b the length of the rectangle

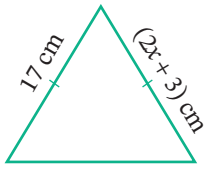
c the width of the rectangle.

9 The profit, $\$P$, made by a hairdresser is given by $P = 18x - 900$, where x represents the number of customers. Find: PS R

a the profit made with 195 customers

b the number of customers if the profit is $3060.

10 Find the value of x in this isosceles triangle. R



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Extension: Equations and formulas

10.07

YEAR 9 STAGE 5.2

A **formula** is a rule written in algebraic form. It shows the relationship between variables.

For example:

- the formula $A = lw$ gives the area, A, of a rectangle with length l and width w.
- the formula $D = ST$ gives the distance, D, travelled by a car at a speed S for time T.

Solving mathematical problems often involves substituting values in formulas and solving equations.

WS Working with formulas

Example 9

The formula for the perimeter, P, of a rectangle of length l and width w is given by $P = 2(l + w)$. Find a rectangle's:

a length if its perimeter is 40 cm and its width is 8 cm

b width if its perimeter is 100 cm and its length is 35 cm.

Solution

a Substitute $P = 40$, $w = 8$:

$$P = 2(l + w)$$
$$40 = 2(l + 8)$$
$$40 = 2l + 16$$
$$2l + 16 = 40$$
$$2l + 16 - 16 = 40 - 16$$
$$2l = 24$$
$$\frac{2l}{2} = \frac{24}{2}$$
$$l = 12$$

b Substitute $P = 100$, $l = 35$:

$$P = 2(l + w)$$
$$100 = 2(35 + w)$$
$$100 = 70 + 2w$$
$$70 + 2w = 100$$
$$70 + 2w - 70 = 100 - 70$$
$$2w = 30$$
$$\frac{2w}{2} = \frac{30}{2}$$
$$w = 15$$

EXERCISE 10.07 ANSWERS ON P. 577

Equations and formulas U F PS R C

EXAMPLE 9

1 The volume of a rectangular prism with length, l, width, w, and height, h, is given by $V = lwh$. Use this formula to find the length of a rectangular prism with width 4 and height 5 if its volume is 340. Select the correct answer **A**, **B**, **C** or **D**.

A 17 **B** 68 **C** 85 **D** 425

2 The number of toothpicks (T) needed to build a row of N squares is $T = 3N + 1$.

a How many toothpicks are needed to build a row of 4 squares?

b How many squares can be built using 322 toothpicks?

Foundation Standard Complex

YEAR 9
STAGE 5.2

3 The number of hours (H) of sleep that children need depends on their age (A) in years and is given by the formula $H = 17 - \frac{A}{2}$. Find: PS R

a the hours of sleep needed when a child is 14 years old

b how old a child who needs 8 hours sleep is.

4 Using the perimeter formula for a rectangle, $P = 2(l + w)$, find: PS R

a the perimeter when length = 13 and width = 4

b the length when perimeter = 36 and width = 7

c the width when perimeter = 58 and length = 16

5 The area of a trapezium is given by $A = \frac{1}{2}(a+b)h$, where a and b are the lengths of its parallel sides and h is the perpendicular height between them. Find the value of: PS R

a the area of a trapezium with parallel sides 20 and 13 and height 8

b the height of a trapezium with parallel sides 22 and 8 and area 90

c a parallel side of a trapezium with area 64, height 4 and other parallel side 17

d a parallel side of a trapezium with other parallel side 5, height 12 and area 120

6 The volume of a pyramid is given by $V = \frac{1}{3}Ah$, where A is the area of the base and h is the height of the pyramid. Find the value of: PS R

a the volume of a pyramid with base area 25 and height 7

b the base area of a pyramid with volume 100 and height 10

c the height of a pyramid with volume 225 and base area 75

7 The simple interest (I) earned on principal \$$P$ invested for n years at an interest rate of r p.a. (where r is a decimal) is given by $I = Prn$. PS R

p.a. means 'per year'

a Find the simple interest earned when \$1250 is invested at 2% p.a. for 5 years (convert 2% to a decimal first).

b Find the principal invested if interest was \$60 and the interest rate was 5% p.a. for 3 years.

c Find the interest rate (as a percentage) if \$700 invested for 3 years earned \$84 in interest.

d Find the number of years that \$1650 was invested if it earned \$346.50 interest at 3% p.a.

8 **a** 3 people meet and each person shakes hands with every other person. How many handshakes were there? R

b In how many ways can 4 people shake hands?

c The formula $H = \frac{P(P-1)}{2}$, where H is the number of handshakes, and P is the number of people, gives the number of shakes between people. In how many ways can 10 people shake hands?

d How many people are needed for 105 handshakes?

Power plus ANSWERS ON P. 577

1 Solve each equation.

a $2(x+1)+2(x-1)=12$

b $2(x+4)-3(x-1)=9$

c $4(2x-1)-5(x-2)=6$

d $2-(3x+5)=4(x+1)$

e $\frac{x+7}{4}=\frac{6(x-1)}{3}$

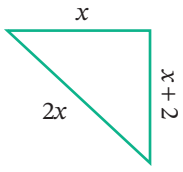
f $\frac{2(x+1)}{3}=\frac{5(x-2)}{2}$

g $\frac{2x}{3}-\frac{x}{6}=10$

h $\frac{3x}{4}+\frac{9x}{10}=44$

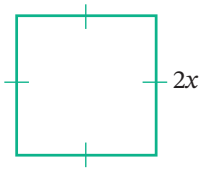
2 For each diagram, write an equation for x and solve it.

a



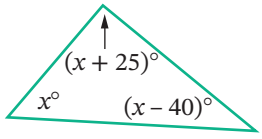
Perimeter = 30

b



Area = 100

c



3 Solve each quadratic equation.

a $4x^2=100$

b $5x^2-7=73$

c $\frac{2x^2}{3}=24$

d $3x^2+8=20$

e $(x+4)^2=9$

f $x^2+2x=15$

4 Diophantus was a famous mathematician who was the first to abbreviate his mathematical thoughts using symbols. He is known as the Greek father of algebra and when he died, one of his admirers wrote the following riddle about his life:

Diophantus' youth lasted $\frac{1}{6}$ of his life. He grew a beard after $\frac{1}{12}$ more. After $\frac{1}{7}$ more of his life, Diophantus married; 5 years later he had a son. The son lived exactly $\frac{1}{2}$ as long as his father, and Diophantus died just 4 years after his son. All this adds to the years Diophantus lived.

Write this riddle as an equation and solve it to find how long Diophantus lived. (*Hint*: Let x years equal his life.)

CHAPTER 10 REVIEW

Language of maths

Solving equations

backtracking	balancing	brackets	check
equation	expand	formula	inverse operation
LHS (left-hand side)	one-step equation	pronumeral	quadratic equation
RHS (right-hand side)	solution	solve	substitution
two-step equation	undoing	unknown	variable

1 What are the 2 algebraic methods for solving equations?

2 Which method involves 'undoing' operations?

3 What does the word 'solution' mean?

4 Which word in the list means 'opposite'?

5 Why is the **variable** in an equation sometimes called an **unknown**?

6 What word means to rewrite an algebraic expression by removing the brackets?

7 What does **RHS** stand for:

a in congruent triangles?

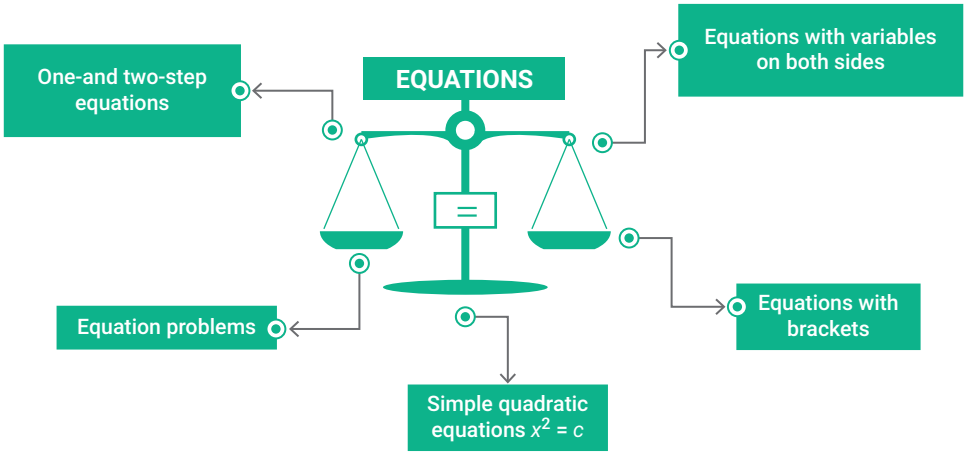
b in solving equations?

Topic summary

Mind map: Equations

- Which parts of this topic did you find easy? What did you already know?
- Give examples of some problems that might be solved using equations.
- Are there any parts of this topic that you still don't understand? Talk to your teacher about them.
- In what sort of careers would people use equations?

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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TEST YOURSELF 10

ANSWERS ON P. 577

1 Solve each equation. 10.01

a $k + 6 = 13$ **b** $x - 3 = 8$ **c** $a + 3 = 17$

d $a - 12 = 21$ **e** $3x = 12$ **f** $10f = 120$

g $\frac{m}{4} = 7$ **h** $\frac{x}{3} = 8$ **i** $w + 9 = 3$

j $k - 5 = -7$ **k** $-2m = -16$ **l** $\frac{x}{3} = -12$

2 Solve each equation. 10.02

a $4p + 3 = 23$ **b** $3m + 17 = 8$ **c** $2x - 12 = 18$

d $\frac{h}{7} + 5 = 16$ **e** $\frac{n}{2} - 8 = -12$ **f** $\frac{a}{5} - 8 = 4$

g $\frac{2y}{5} = 4$ **h** $\frac{x+2}{3} = 4$ **i** $\frac{n-5}{2} = 10$

j $\frac{5x}{2} = -15$ **k** $\frac{k+9}{7} = 5$ **l** $\frac{d-1}{5} = -6$

3 Solve each equation. 10.03

a $3x + 4 = x + 6$ **b** $5u - 3 = 2u + 6$ **c** $12h - 8 = 8h + 4$

d $3v - 4 = 7v + 8$ **e** $2x + 9 = 7x - 4$ **f** $9 - 5t = 3t - 15$

4 Solve each equation. 10.04

a $2(n + 3) = 18$ **b** $3(x + 1) = 15$ **c** $10(x - 3) = -10$

d $5(x - 2) = 3x + 4$ **e** $4(y + 1) = y + 18$ **f** $-2(d - 2) = 2d - 20$

g $3(r - 1) = 2(r + 9)$ **h** $7(a + 5) = 3(a + 9)$ **i** $2(2n - 4) = 2(5 - n)$

5 Solve each equation: 10.05

a $x^2 = 64$

b $x^2 = 45$, correct to one decimal place

c $5x^2 = 135$, as a surd

6 **a** If a number is doubled and has 4 added to it, the answer is 14. What is the number? 10.06

b If a number has 4 added to it and is doubled, the answer is 14. What is the number?

7 **a** Keilani is paid \$45 for each jumper she knits. If n is the number of jumpers Keilani knits to earn a total of \$270, which equation can be used to find the value of n? Select the correct answer **A**, **B**, **C** or **D**. 10.06

A $\frac{n}{45} = 270$ **B** $45n = 270$ **C** $n + 45 = 270$ **D** $270 - n = 45$

b Solve the equation to find the value of n.

8 7 times a number is the same as 9 more than 4 times the same number. Use an equation to find the number. 10.06

9 The sum (S) of the angles (in degrees) of a polygon is given by $S = 180(n - 2)$, where n is the number of sides. Find: 10.06

a the sum of the interior angles when a polygon has 9 sides

b the number of sides in a polygon whose angle sum is 1080°.

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